



Munich Personal RePEc Archive

Second-Degree Price Discrimination on Two-Sided Markets

Enrico Böhme

30. August 2012

Online at <https://mpra.ub.uni-muenchen.de/40951/>

MPRA Paper No. 40951, posted 30. August 2012 09:20 UTC

Second-Degree Price Discrimination on Two-Sided Markets

Enrico Böhme*

Johann Wolfgang Goethe-University, Frankfurt

August 2012

Abstract

The present paper provides a descriptive analysis of the second-degree price discrimination problem on a monopolistic two-sided market. By imposing a simple two-sided framework with two distinct types of agents on one of its market sides, it will be shown that under incomplete information, the extent of platform access for high-demand agents is strictly reduced below the benchmark level (complete information). In addition, the paper's findings imply that it is feasible in the optimum to charge higher payments from low-demand agents if the extent of interaction with agents from the opposite market side is assumed to be bundle-specific.

Keywords: two-sided markets, second-degree price discrimination, monopoly

JEL Classification: D42, D82, L12, L15

* Chair of Public Finance, Faculty of Economics and Business Administration, Goethe University Frankfurt, Grüneburgplatz 1, D-60323 Frankfurt am Main, Germany. E-mail: boehme@econ.uni-frankfurt.de. I highly appreciate the helpful comments and hints of Christopher Müller. All remaining mistakes are my responsibility.

1. Introduction

Second-degree price discrimination is a well-known phenomenon in the field of Industrial Organization, since it is present in many industries. For instance, non-linear pricing schemes are very common in the telecommunications industry, in insurance markets, or in railroad and airline industries. The corresponding problem of a monopolistic firm seeking to maximize profit by offering type-specific bundles that are voluntarily chosen by the appropriate type of consumer has been widely analyzed in the economic literature. Seminal papers are Spence (1977), Stiglitz (1977), Mussa and Rosen (1978), Maskin and Riley (1984), or Spulber (1993). More recently, second-degree price discrimination has also been discussed in the context of duopolistic competition, yielding ambiguous effects of price discrimination on profits. The most relevant paper dealing with this topic is Stole (1995), whereas Armstrong (2006a) and Stole (2007) survey this literature.

The economic literature mentioned above refers to the case of traditional one-sided markets, while many industries operate on two-sided markets, i.e. markets where platforms enable interaction between two distinct groups of agents. Examples for two-sided networks are manifold: Real estate agencies facilitate interaction between house buyers and sellers, credit card companies establish a simple way of payments between consumers and merchants, whereas media platforms allow advertisers to interact with media consumers. The economics of two-sided markets has been extensively analyzed in the economic literature. While Roche and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006b) analyze monopolistic and duopolistic price-setting behaviour in more general two-sided frameworks, Anderson and Coate (2005), Gabszewicz et al. (2004), Gal-Or and Dukes (2003), and Peitz and Valletti (2008)) specifically focus on media markets.

So far, only little attention has been given to non-linear pricing strategies on two-sided markets. Caillaud and Jullien (2003) and Armstrong (2006b) analyze the case of group-specific prices, i.e. third-degree price discrimination, whereas two-part tariffs are considered by Roche and Tirole (2003), Armstrong (2006b), and Reisinger (2010). In addition, Liu and Serfes (2008) study first-degree price discrimination on a two-sided duopolistic market. To our knowledge, second-degree price discrimination so far has not been analyzed in the context of two-sided networks. However, offering type-specific bundles for different types of agents is very common in two-sided industries. For instance, movie theatres offer multi-ticket bundles or even flat-rates, pay-TV platforms sell different combinations of content and price,

while online dating platforms allow men to subscribe for, e.g., one month, six months, or one year with a decreasing price per month. Recently, second-degree price discrimination became particularly relevant in the newspaper industry as many newspaper companies started to additionally offer their content on the internet. Here, different strategies can be observed. While some companies simply offer identical content via pay-per-view access, others publish reduced content, e.g. shortened articles or original articles with a significant time delay, without charging any price, but exposing readers to advertising. Compared to the traditional printed newspaper, this may well be interpreted as a different bundle that contains quality-reduced content for a lower price. Surprisingly, it can also be observed that some newspapers, e.g. Germany's best-selling newspaper "Bild", regularly publish exclusive print media content (e.g. exclusive stories or soccer trade rumors) without any quality reduction on the internet free of charge, generating revenues from advertising only.

It is the aim of this paper to make a first step in analyzing second-degree price discrimination on monopolistic two-sided markets. Tailored around the examples mentioned above, we will develop a simple framework with asymmetric information on one of its two market sides. This specific side of the market is supposed to consist of two distinct types of agents with different valuations regarding the intrinsic utility they obtain from joining the platform. As per usual, the agents' utility on either side of the market is also affected from indirect network externalities.

We will show that many of the well-known results from second-degree price discrimination on one-sided markets still prevail in our two-sided framework. However, in contrast to the "no-distortion-at-the-top" result from one-sided markets, we find that due to the two-sidedness of the market, the profit-maximizing quantity for high-demand agents is strictly reduced under incomplete information. In addition, our findings indicate that if the interaction with agents from the opposite market side depends on the chosen bundle, it is a feasible optimal solution that the bundle for low-demand agents is more expensive than the bundle for high-demand agents.

The paper is organized as follows: In Section 2, we will develop the analytical framework, whereas Section 3 analyzes the price setting behaviour of a monopolistic platform operator. In this context we will discuss the benchmark case of complete information and compare our results to the case of incomplete information. Section 4 follows the same structure, but imposes bundle-specific interaction, i.e. the extent of interaction will depend on the chosen

bundle. In Section 5, we will summarize our findings and the contribution of our paper. In addition, we will suggest directions for further research.

2. Analytical Framework

In the following section, we will develop a benchmark model of a monopolistic platform operator that operates on a two-sided market. Our theoretical framework considers two market sides $k = 1, 2$, where market side 1 consists of two distinct groups of agents, labelled H and L. Agents of both groups differ in their intrinsic valuation for joining the platform. While the agents know their individual type, the platform operator is not able to distinguish agents with respect to their type. Hence, except for the benchmark case, this situation is characterized by asymmetric information.

The agents' utility is supposed to consist of two elements: an intrinsic utility that depends on each agent's access to the platform, denoted by n_1 , as well as an indirect network effect from the presence of market side 2 agents and the total payment t . The corresponding utility function is assumed to be additive-separable and can be described by

$$U_i(n_1^i, n_2, t_i) = \theta_i \cdot u(n_1^i) - \alpha \cdot n_2(\cdot) - t_i \quad i = H, L,$$

where $u(\cdot)$ represents the utility from joining the platform, θ reflects the type-dependent valuation, and α denotes the indirect network externality resulting from the presence of n_2 market side 2 agents. For $\alpha > 0$, the network externality is negative, while $\alpha < 0$ implies an additional benefit for agents on market side 1 from interacting with agents from the opposite market side. In addition, we assume $u'(\cdot) > 0$, $u''(\cdot) < 0$ and $\theta_H > \theta_L$, whereas the absolute number of agents in each group is set to one. The latter assumption implies that our results will be independent from the type distribution of agents, which allows for an analysis that is strictly focused on the effects resulting from extending the problem from one-sided markets to two-sided markets.

Market side 2 is characterized by a traditional downward sloping demand function $n_2(n_1^i, p)$, where p denotes the price that all agents on this market side have to pay, in order to join the platform. Since we assumed that the absolute number of agents on market side 1 is fixed, agents on market side 2 do not care about the total number of agents from market side 1, but about the extent of their access to the platform. This implies that a potential benefit from

interaction is assumed to depend on the extent of the interaction process.¹ If the externality is positive, we have $\frac{\partial n_2}{\partial n_1^L} > 0$, while a negative indirect network effect implies $\frac{\partial n_2}{\partial n_1^L} < 0$. As the demand function was supposed to be downward sloping in prices, we assume that $\frac{\partial n_2}{\partial p} < 0$. Additional assumptions are $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H}$, and the existence of a unique interior solution, i.e. it is supposed that both types of agents on market side 1 are served in the optimum.

Reservation utility of market side 1 agents is supposed to be $\bar{U}_i = 0$. The cost function of the monopolistic platform operator is assumed to consist of constant marginal cost, c , for each unit of access to market side 1. In order to simplify the analysis, total costs on market side 2 are assumed to be equal to zero. The resulting cost function is given by

$$C(n_1^L, n_1^H) = c \cdot n_1^L + c \cdot n_1^H.$$

Our theoretical framework refers to the extent of platform access that is sold to agents on market side 1, which may well be interpreted as being equivalent to selling different quantities of a consumption good to different types of consumers. This allows for a comparison of our results to the well-known second-degree price discrimination outcome from traditional one-sided markets.

3. Model Analysis

The benchmark case of complete information

Under complete information, the monopolist maximizes profits by selling a type-specific bundle of platform access and payment to each type of agents on market side 1. It must be taken into account that both types must be willing to accept their offer (participation constraints). Hence, the maximization problem can be described by

$$(1) \quad \max_{t_L, t_H, n_1^L, n_1^H, p} \Pi = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p)$$

s.t.

$$(1a) \quad \theta_L \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0,$$

$$(1b) \quad \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq 0.$$

Obviously, the participation constraints have to be binding in the optimum as the platform operator is able to exploit the entire consumer surplus. Therefore, t_L and t_H in (1) can be substituted by (1a) and (1b), which leads to the Lagrangian

¹ For instance, on media markets this assumption corresponds to the well-known concept of “persuasive advertising”.

$$(2) \quad \max_{n_1^L, n_1^H, p} L = \theta_L \cdot u(n_1^L) - \alpha \cdot n_2(\cdot) + \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(\cdot) - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(\cdot)$$

with the first-order conditions

$$(3) \quad \frac{\partial L}{\partial n_1^L} = \theta_L \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(4) \quad \frac{\partial L}{\partial n_1^H} = \theta_H \cdot \frac{\partial u}{\partial n_1^H} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(5) \quad \frac{\partial L}{\partial p} = \underbrace{n_2(\cdot)}_{>0} + \underbrace{(p - 2 \cdot \alpha)}_{>0(!)} \cdot \underbrace{\frac{\partial n_2}{\partial p}}_{<0} = 0.$$

Since $n_2(\cdot) > 0$ and $\frac{\partial n_2}{\partial p} < 0$, we can immediately conclude from equation (5) that an interior solution requires $(p - 2 \cdot \alpha) > 0$. Then, respecting the assumptions specified in Section 2, equations (3) to (5) implicitly define the unique interior solution $(n_1^{L*}, n_1^{H*}, p^*)$.

As per usual, the monopolist's profit is maximized where marginal profit is equal to zero. However, it is not surprising that our results are more complex than the analog outcome on one-sided markets as the marginal profit also accounts for the arising network externalities. Since we know that $(p - 2 \cdot \alpha) > 0$, it is obvious that in case of $\frac{\partial n_2}{\partial n_1^i} > 0$, $i = H, L$, a marginal increase in n_1^i generates additional profit, whereas for $\frac{\partial n_2}{\partial n_1^i} < 0$ each additional unit of n_1^i has a negative impact on marginal profit.

Comparing equations (3) and (4), while taking into account that $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H}$ and $\theta_H > \theta_L$, it is easy to verify that the marginal profit from an additional unit of access for the H-type, n_1^H , strictly exceeds the marginal profit from an increase in n_1^L . Hence, we know that in the optimum $n_1^{H*} > n_1^{L*}$ must hold, which is in line with the corresponding result from one-sided markets. As we know that (1a) as well as (1b) are binding in the optimum, profit-maximizing tariffs are given by

$$t_L^* = \theta_L \cdot u(n_1^{L*}) - \alpha \cdot n_2(n_1^{L*}, n_1^{H*}, p^*),$$

$$t_H^* = \theta_H \cdot u(n_1^{H*}) - \alpha \cdot n_2(n_1^{L*}, n_1^{H*}, p^*).$$

Therefore, respecting that $\theta_H > \theta_L$ and $n_1^{H*} > n_1^{L*}$, we can conclude that

$$t_H^* = \theta_H \cdot u(n_1^{H*}) - \alpha \cdot n_2(n_1^{L*}, n_1^{H*}, p^*) > t_L^* = \theta_L \cdot u(n_1^{L*}) - \alpha \cdot n_2(n_1^{L*}, n_1^{H*}, p^*).$$

Obviously, under complete information there is no qualitative difference in the results of our two-sided markets model when compared to the corresponding outcome of the second-degree price discrimination problem on one-sided markets: The type-specific bundle for the H-type contains more platform access and a higher payment than the one for the L-type.

The case of incomplete information

In the case of asymmetric information, the monopolistic platform operator is not able to distinguish the type of market side 1 agents. Hence, it must be taken into account that each type of agent on market side 1 must be willing to voluntarily choose its designated bundle (incentive constraints). With incentive and participation constraints, the corresponding optimization problem is given by

$$\begin{aligned}
 (6) \quad & \max_{t_L, t_H, n_1^L, n_1^H, p} \Pi = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p) \\
 & \text{s.t.} \\
 (6a) \quad & \theta_L \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0, \\
 (6b) \quad & \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq 0, \\
 (6c) \quad & \theta_L \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq \theta_L \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H, \\
 (6d) \quad & \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq \theta_H \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L.
 \end{aligned}$$

Using equations (6a) and (6d) as well as $\theta_H > \theta_L$, we find that

$$\begin{aligned}
 & \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq \theta_H \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \\
 & > \theta_L \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0 \Rightarrow \theta_H \cdot u(n_1^H) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H > 0,
 \end{aligned}$$

which implies that the participation constraint for the H-type is never binding. Hence, this restriction can be ignored with respect to the optimization process, so that the resulting Kuhn-Tucker problem is formally described by

$$\begin{aligned}
 (7) \quad & \max_{t_L, t_H, n_1^L, n_1^H, p, \lambda_1, \lambda_2, \lambda_3} L = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p) \\
 & + \lambda_1 \cdot [\theta_L \cdot u(n_1^L) - \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L] + \lambda_2 \cdot [\theta_L \cdot u(n_1^L) - t_L - \theta_L \cdot u(n_1^H) + t_H] \\
 & + \lambda_3 \cdot [\theta_H \cdot u(n_1^H) - t_H - \theta_H \cdot u(n_1^L) + t_L],
 \end{aligned}$$

leading to the first-order conditions

$$(8) \quad \frac{\partial L}{\partial t_L} = 1 - \lambda_1 - \lambda_2 + \lambda_3 = 0,$$

$$(9) \quad \frac{\partial L}{\partial t_H} = 1 + \lambda_2 - \lambda_3 = 0,$$

$$(10) \quad \frac{\partial L}{\partial n_1^L} = (\lambda_1 + \lambda_2) \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \lambda_3 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^L} + (p - \lambda_1 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(11) \quad \frac{\partial L}{\partial n_1^H} = \lambda_3 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} - \lambda_2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^H} + (p - \lambda_1 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(12) \quad \frac{\partial L}{\partial p} = n_2(\cdot) + (p - \lambda_1 \cdot \alpha) \cdot \frac{\partial n_2}{\partial p} = 0.$$

Since the Kuhn-Tucker conditions require $\lambda_m \geq 0$, $m = 1, 2, 3$, we find from equations (8) and (9) that a solution is characterized by $\lambda_1 = 2$, $\lambda_2 = 0$, and $\lambda_3 = 1$. Therefore, the remaining first-order conditions become

$$(13) \quad \frac{\partial L}{\partial n_1^L} = 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(14) \quad \frac{\partial L}{\partial n_1^H} = \theta_H \cdot \frac{\partial u}{\partial n_1^H} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(15) \quad \frac{\partial L}{\partial p} = \underbrace{n_2(\cdot)}_{>0} + \underbrace{(p - 2 \cdot \alpha)}_{>0(!)} \cdot \underbrace{\frac{\partial n_2}{\partial p}}_{<0} = 0.$$

Equation (15) immediately implies that an interior solution still requires $(p - 2 \cdot \alpha) > 0$ as was already the case under complete information. Assuming that $(p - 2 \cdot \alpha) > 0$ holds, the unique interior solution $(n_1^{L**}, n_1^{H**}, p^{**})$ of the maximization problem under incomplete information is implicitly described by the equation system (13) to (15).

Since $\theta_H > \theta_L$, it is easy to show that

$$(16) \quad 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} = (2 \cdot \theta_L - \theta_H) \frac{\partial u}{\partial n_1^L} < \theta_L \cdot \frac{\partial u}{\partial n_1^L}.$$

As we have additionally assumed that $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H}$, we know from comparing equations (13) and (14) that $\frac{\partial L}{\partial n_1^H} > \frac{\partial L}{\partial n_1^L}$. Therefore, we can conclude that $n_1^{H**} > n_1^{L**}$ must hold in the optimum, so that the monopolist still offers a higher extent of platform access to the high-demand agents. Interpreting the Kuhn-Tucker conditions shows that the participation constraint for the L-type and the incentive constraint for the H-type are binding in the optimum. Using (6a) and (6d), the optimal payments are therefore given by

$$\begin{aligned} t_L^{**} &= \theta_L \cdot u(n_1^{L**}) - \alpha \cdot n_2(n_1^{L**}, n_1^{H**}, p^{**}) \\ t_H^{**} &= \theta_H \cdot u(n_1^{H**}) - \theta_H \cdot u(n_1^{L**}) + t_L^{**}, \end{aligned}$$

which immediately implies that

$$t_H^{**} - t_L^{**} = \theta_H \cdot u(n_1^{H**}) - \theta_H \cdot u(n_1^{L**}) = \theta_H \cdot \underbrace{(u(n_1^{H**}) - u(n_1^{L**}))}_{>0} > 0.$$

Obviously, under incomplete information we still find that the bundle for the H-type contains more platform access and a higher payment compared to the bundle for the L-type. In addition, we can conclude from the Kuhn-Tucker conditions that the L-type's consumer surplus in the optimum is equal to zero, while the H-type enjoys a strictly positive consumer surplus. Since (6d) is binding in the optimum, we know that the H-type is indifferent between buying the bundles offered. The results obtained so far are entirely consistent with the corresponding findings on one-sided markets.

Comparing the outcomes

The final step in analyzing the model is to compare the results under incomplete information to the benchmark case of complete information. On one-sided markets this comparison produces the well-known “no-distortion-at-the-top” rule, i.e. one finds that in case of assymetric information, the bundle for low-demand consumers contains less quantity than under complete information, while high-demand consumers are provided with the efficient quantity. The corresponding findings of our two-sided market model are summarized in Proposition 1:

Proposition 1: Under incomplete information, the profit-maximizing amount of platform access for L-type agents is strictly smaller than under complete information, while the optimal level of platform access for H-type agents is also strictly below the benchmark level, which contradicts the findings from one-sided markets. This result is independent of the sign of the network externality exerted on market side 2 agents.

Proof: First, we consider the case of positive indirect network externalities on market side 2, i.e. $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} > 0$. Using (16) as well as equations (3) and (13), we find that

$$(17) \quad \theta_L \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c > 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c,$$

which implies that the optimal amount of platform access for low-demand agents on market side 1 is ceteris paribus smaller under incomplete information than under complete information. This effect has an additional impact on the demand of market side 2 agents, which in turn influences the profit-maximizing price that is charged on market side 2. The optimal prices p^* and p^{**} were implicitly determined by equations (5) and (15). Differentiating (5) or (15) with respect to n_1^L and additionally assuming $\frac{\partial^2 n_2}{\partial p \partial n_1^L} = 0$ yields

$$\frac{\partial^2 L}{\partial p \partial n_1^L} = \underbrace{\frac{\partial n_2(\cdot)}{\partial n_1^L}}_{>0} + (p - 2 \cdot \alpha) \cdot \underbrace{\frac{\partial^2 n_2}{\partial p \partial n_1^L}}_{=0} > 0,$$

which allows us to conclude that $p^{**} < p^*$, so that the optimal price on market side 2 is strictly smaller than in the benchmark case of complete information. Since p simultaneously enters equation (13), it follows that the marginal profit from an increase in n_1^L is additionally reduced under incomplete information. Hence, we find that $n_1^{L**} < n_1^{L*}$. At the same time p also enters equation (14), which leads to a decrease in the marginal profit generated from an additional unit n_1^H . Therefore, we obtain $n_1^{H**} < n_1^{H*}$.

If we consider the case of $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} < 0$, we find that (17) still holds, which again implies that n_1^L is ceteris paribus reduced under incomplete information. Due to the negative network externality on market side 2, this effect leads to an increase in the demand of market side 2 agents. Assuming $\frac{\partial^2 n_2}{\partial p \partial n_1^L} = 0$, the corresponding impact on p^{**} is described by

$$\frac{\partial^2 L}{\partial p \partial n_1^L} = \underbrace{\frac{\partial n_2(\cdot)}{\partial n_1^L}}_{<0} + (p - 2 \cdot \alpha) \cdot \underbrace{\frac{\partial^2 n_2}{\partial p \partial n_1^L}}_{=0} < 0,$$

which obviously implies that $p^{**} > p^*$. Using (13) and (14), and respecting that an interior solution requires $(p - 2 \cdot \alpha) > 0$, it follows that $n_1^{L**} < n_1^{L*}$ as well as $n_1^{H**} < n_1^{H*}$.

(q.e.d.)

The economic intuition behind our findings is straightforward: The monopolistic platform operator is maximizing her profit by inducing a self-selection process among both types of agents on market side 1. Therefore, the bundle of platform access and payment that is sold to low-demand agents ceteris paribus contains less platform access than under complete information as this allows the platform operator to extract additional consumer surplus from high-demand agents. This result is well-known from second-degree price discrimination on one-sided markets. However, the reduction of n_1^L induces an additional effect on the opposite market side as the demand of market side 2 agents was supposed to depend on n_1^L and n_1^H . For instance, assume a positive network externality exerted on market side 2. Then, a reduction of n_1^L shifts the demand function $n_2(n_1^L, n_1^H, p)$ inwards, which reduces the optimal price p^{**} that is charged on market side 2. This in turn makes a marginal increase in n_1^L even less profitable, so that the profit-maximizing extent of platform access for low-demand agents is strictly smaller under incomplete information. In addition, the reduction of p^{**} also affects n_1^{H**} as each marginal unit of platform access for high-demand agents generates less marginal profit than under complete information. This effect strictly reduces n_1^{H**} below the benchmark level, which contradicts the corresponding results from one-sided markets, where the quantity for high-demand consumers was not affected under incomplete information.

The outcome of the second-degree price discrimination problem on one-sided markets may also serve as an alternative benchmark for a comparison with our results under incomplete information. The case of one-sided markets is equivalent to the special case of our model where $\alpha = 0$ and $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} = 0$. In this special case, equations (13) and (14) become

$$\begin{aligned}\frac{\partial L}{\partial n_1^L} &= 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot 0) \cdot 0 - c = 0 \Leftrightarrow \theta_L \cdot \frac{\partial u}{\partial n_1^L} > c, \\ \frac{\partial L}{\partial n_1^H} &= \theta_H \cdot \frac{\partial u}{\partial n_1^H} + (p - 2 \cdot 0) \cdot 0 - c = 0 \Leftrightarrow \theta_H \cdot \frac{\partial u}{\partial n_1^H} = c,\end{aligned}$$

which reproduces the “no-distortion-at-the-top” outcome from one-sided markets. We will denote this specific solution by $(\hat{n}_1^L, \hat{n}_1^H)$.

If we consider the case of $\alpha \neq 0$ and $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} > 0$, while additionally respecting $(p - 2 \cdot \alpha) > 0$, we find by using (13) and (14) that

$$\begin{aligned}\frac{\partial L}{\partial n_1^L} &= 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} + (p - 2 \cdot \alpha) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0 \Leftrightarrow \theta_L \cdot \frac{\partial u}{\partial n_1^L} + \underbrace{(p - 2 \cdot \alpha)}_{>0} \cdot \underbrace{\frac{\partial n_2}{\partial n_1^L}}_{>0} > c, \\ \frac{\partial L}{\partial n_1^H} &= \theta_H \cdot \frac{\partial u}{\partial n_1^H} + \underbrace{(p - 2 \cdot \alpha)}_{>0} \cdot \underbrace{\frac{\partial n_2}{\partial n_1^H}}_{>0} - c = 0 \Leftrightarrow \theta_H \cdot \frac{\partial u}{\partial n_1^H} < c,\end{aligned}$$

which implies that $n_1^{L**} > \hat{n}_1^L$ as well as $n_1^{H**} > \hat{n}_1^H$. It is easy to verify that for $\alpha \neq 0$ and $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} < 0$, we find the opposite result, that is $n_1^{L**} < \hat{n}_1^L$ and $n_1^{H**} < \hat{n}_1^H$.

The economic intuition behind our results is rather simple: In case of positive network externalities on market side 2, i.e. $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} > 0$, each additional unit of n_1^L and n_1^H generates additional marginal profit. Hence, the monopolist strictly increases the amount of platform access in both bundles compared to the solution under asymmetric information on one-sided markets. In the optimum, we find that the H-type’s marginal willingness to pay for n_1^{H**} is strictly below marginal cost, which implies that high-demand agents on market side 1 are subsidized in any case. Under specific parameter sets, the same even holds for low-demand agents. Considering the case of $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} < 0$, we find the opposite: A marginal increase in n_1^L and n_1^H leads to additional marginal cost, thus inducing the monopolistic platform operator to strictly reduce the extent of platform access sold to each type of agents on market side 1.

4. A Model with bundle-specific Interaction

So far, we have assumed that both types of agents on market side 1 are equivalently affected by the presence of market side 2 agents. In this section, we will extend the analysis by

including bundle-specific interaction, i.e. the extent of the indirect externality depends on the chosen bundle. For instance, consider an online dating platform that sells different pairs of platform access and payments to two distinct types of market side 1 agents (men) with different valuations, while market side 2 agents (women) are generally allowed to join the platform for free. Then, it could be the case that the L-type bundle only allows market side 1 agents to contact a limited number of agents from the opposite market side, whereas the bundle for high-demand agents contains unlimited access.

We will account for this by imposing a different utility function for agents on market side 1. This function is still supposed to be additive separable and takes the form

$$U_i(n_1^i, n_2, t_i) = \theta_i \cdot u(n_1^i) - n_1^i \cdot \alpha \cdot n_2(\cdot) - t_i \quad i = H, L,$$

while the notation introduced in Section 2 remains the same. In addition, all assumptions made so far are still valid. The motivation behind this model specification stems from the movie theatre example mentioned in Section 1. Assume that a cinema operator offers two bundles: The L-type bundle contains a single ticket for a specific price, while the H-type bundle includes 5 tickets for a total payment that implies a per ticket price below the price of the single ticket. If both types of consumers buy their designated bundles, the L-type is only once exposed to the cinema-specific amount of advertising, while the H-type watches the same amount of advertising five times.

Profit maximization under complete information

In case of symmetric information, the monopolistic platform operator is again able to perfectly discriminate between both types of agents on market side 1 by selling type-specific bundles (n_1^i, n_2, t_i) , as take-it-or-leave-it offers. Respecting the participation constraints, the corresponding optimization problem is given by

$$(18) \quad \max_{t_L, t_H, n_1^L, n_1^H, p} \Pi = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p)$$

s.t.

$$(18a) \quad \theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0,$$

$$(18b) \quad \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq 0.$$

As we have already mentioned in Section 2, perfect price discrimination implies that the consumer surplus for market side 1 agents is equal to zero. Hence, we know that the participation constraints are binding in the optimum, so that the maximization problem simplifies to a Lagrangian of the form

$$(19) \quad \max_{n_1^L, n_1^H, p} L = \theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(\cdot) + \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(\cdot) \\ - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(\cdot),$$

yielding the first-order conditions

$$(20) \quad \frac{\partial L}{\partial n_1^L} = \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \alpha \cdot n_2(\cdot) + (p - \alpha \cdot (n_1^L + n_1^H)) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(21) \quad \frac{\partial L}{\partial n_1^H} = \theta_H \cdot \frac{\partial u}{\partial n_1^H} - \alpha \cdot n_2(\cdot) + (p - \alpha \cdot (n_1^L + n_1^H)) \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(22) \quad \frac{\partial L}{\partial p} = \underbrace{n_2(\cdot)}_{>0} + \underbrace{(p - \alpha \cdot (n_1^L + n_1^H))}_{>0(!)} \cdot \underbrace{\frac{\partial n_2}{\partial p}}_{<0} = 0.$$

Since all assumptions from the previous sections are still holding, we can conclude from equation (22) that a feasible solution requires $(p - \alpha \cdot (n_1^L + n_1^H)) > 0$. Assuming that an interior solution exists, this solution is implicitly characterized by equations (20) – (22) and will be denoted by $(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)$.

Given our extension of bundle-specific interaction, it is not surprising that the effects resulting from the presence of a second market side are more complex than in the benchmark case of Section 3. Analysing equations (20) and (21) reveals that the first-order conditions reflect three different effects (apart from the traditional impact on the marginal willingness to pay): First, a marginal change of n_1^i directly affects the demand of market side 2 agents, which in turn has an indirect impact on the marginal profit generated on this market side. In addition, a different number of market side 2 agents influences the utility of both types of market side 1 agents and hence their willingness to pay. These two (indirect) effects are reflected by the term $(p - \alpha \cdot (n_1^L + n_1^H)) \cdot \partial n_2 / \partial n_1^L$ and are in line with our findings from the benchmark case of Section 3. However, in case of bundle-specific interaction there is an additional effect from a marginal change of n_1^i as type i 's utility on market side 1 is directly affected, because the extent of interaction with agents from the opposite market side changes. This additional effect is reflected by the expression $-\alpha \cdot n_2(\cdot)$.

Since we assumed that $\theta_H > \theta_L$ and $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H}$, it is easy to see by comparing equations (20) and (21) that the marginal profit from an additional unit n_1^H still exceeds the marginal profit from an increase in n_1^L , i.e. $\frac{\partial L}{\partial n_1^H} > \frac{\partial L}{\partial n_1^L}$. Therefore, we know that in the optimum it still holds that $\tilde{n}_1^{H*} > \tilde{n}_1^{L*}$, which corresponds to our findings from the previous section. In addition, we know that the participation constraints are binding in the optimum. Hence, using equations (18a) and (18b), the profit-maximizing tariffs for the type-specific bundles are given by

$$\tilde{t}_L^* = \theta_L \cdot u(\tilde{n}_1^{L*}) - \tilde{n}_1^{L*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*),$$

$$\tilde{t}_H^* = \theta_H \cdot u(\tilde{n}_1^{H*}) - \tilde{n}_1^{H*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*).$$

Therefore, the difference of payments is described by

$$(23) \quad \tilde{t}_H^* - \tilde{t}_L^* = \theta_H \cdot u(\tilde{n}_1^{H*}) - \tilde{n}_1^{H*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*) - \theta_L \cdot u(\tilde{n}_1^{L*}) + \tilde{n}_1^{L*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)$$

$$\Leftrightarrow \tilde{t}_H^* - \tilde{t}_L^* = \theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*}) + \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*) \cdot (\tilde{n}_1^{L*} - \tilde{n}_1^{H*}).$$

So far, our results are not surprising as they are consistent with the results from Section 3. However, analyzing the relation of the bundle-specific payments yields an interesting result, denoted in Proposition 2.

Proposition 2: Under complete information, the profit-maximizing tariff for the H-type bundle strictly exceeds the optimal price-level for the L-type bundle if the indirect network externality exerted on market side 1 agents is positive or absent, i.e. $\alpha \leq 0$. In case of a negative network externality on market side 1, i.e. $\alpha > 0$, it is a feasible profit-maximizing solution that the bundle for low-demand agents is more expensive than the one for high-demand agents, even though the latter contains a strictly higher extent of platform access.

Proof: Suppose $\alpha < 0$. As we know that $\theta_H > \theta_L$ as well as $\tilde{n}_1^{H*} > \tilde{n}_1^{L*}$, we can immediately conclude from equation (23) that

$$\tilde{t}_H^* - \tilde{t}_L^* = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)}_{<0} \cdot \underbrace{(\tilde{n}_1^{L*} - \tilde{n}_1^{H*})}_{<0} > 0.$$

For $\alpha = 0$, equation (23) becomes

$$\tilde{t}_H^* - \tilde{t}_L^* = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*})}_{>0} > 0,$$

which closes the proof for the first part of Proposition 2.

In case of a negative indirect network effect, i.e. $\alpha > 0$, we find by using (23) that

$$\tilde{t}_H^* - \tilde{t}_L^* = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)}_{>0} \cdot \underbrace{(\tilde{n}_1^{L*} - \tilde{n}_1^{H*})}_{<0},$$

which obviously implies that $\tilde{t}_H^* - \tilde{t}_L^*$ can be either positive, negative or equal to zero. In order to prove that $\tilde{t}_H^* - \tilde{t}_L^* < 0$ is feasible in the optimum, we have to show that

$$\underbrace{\theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)}_{>0} \cdot \underbrace{(\tilde{n}_1^{L*} - \tilde{n}_1^{H*})}_{<0} < 0$$

is, at least under specific restrictions, in line with the first-order conditions. Suppose that $\theta_H \approx \theta_L$. Then we have in the optimum that $\tilde{n}_1^{L*} = \tilde{n}_1^{H*} - \varepsilon$, where $\varepsilon > 0$ is assumed to be very small. In this case, it approximately holds that

$$\begin{aligned} \theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*}) &= \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} \cdot |\tilde{n}_1^{L*} - \tilde{n}_1^{H*}| + \theta_L \cdot \frac{\partial u}{\partial n_1^L} \Big|_{\tilde{n}_1^{L*}} \cdot (\tilde{n}_1^{H*} - \tilde{n}_1^{L*}) \\ &= \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} \cdot |\varepsilon| + \theta_L \cdot \frac{\partial u}{\partial n_1^L} \Big|_{\tilde{n}_1^{L*}} \cdot \varepsilon = \varepsilon \left(\theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} + \theta_L \cdot \frac{\partial u}{\partial n_1^L} \Big|_{\tilde{n}_1^{L*}} \right). \end{aligned}$$

Since an optimal solution requires

$$\theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} = \theta_L \cdot \frac{\partial u}{\partial n_1^L} \Big|_{\tilde{n}_1^{L*}},$$

we find by using (23) that $\tilde{t}_H^* - \tilde{t}_L^*$ can be expressed as

$$\tilde{t}_H^* - \tilde{t}_L^* = 2 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} \cdot \varepsilon + \alpha \cdot n_2(\cdot) \cdot (-\varepsilon) = \varepsilon \cdot \left(2 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} - \alpha \cdot n_2(\cdot) \right).$$

Hence, we have that

$$\tilde{t}_H^* - \tilde{t}_L^* < 0 \iff 2 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} - \alpha \cdot n_2(\cdot) < 0 \iff 2 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}} < \alpha \cdot n_2(\cdot).$$

By analyzing the first-order conditions, in particular equation (21), we can conclude that this inequality is satisfied in the optimum, if and only if it is true that

$$(24) \quad \underbrace{\left(\tilde{p}^* - \alpha \cdot (\tilde{n}_1^{L*} + \tilde{n}_1^{H*}) \right)}_{>0} \cdot \underbrace{\frac{\partial n_2}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}}}_{>0(!)} - \underbrace{c}_{>0} > \underbrace{\theta_H \cdot \frac{\partial u}{\partial n_1^H} \Big|_{\tilde{n}_1^{H*}}}_{>0},$$

which obviously only holds for $\frac{\partial n_2}{\partial n_1^H} > 0$.

(q.e.d.)

The results of Proposition 2 are surprising, since they contradict the standard result from one-sided markets where the monopolist in any case charges a higher tariff for the H-type bundle. However, in case of bundle-specific interaction on a two-sided market, the optimization problem is more complex: Considering the case of $\theta_H \approx \theta_L$, we can conclude from (24), that $\tilde{t}_H^* - \tilde{t}_L^* < 0$ requires a specific relation between the term $\left(\tilde{p}^* - \alpha \cdot (\tilde{n}_1^{L*} + \tilde{n}_1^{H*}) \right) \cdot \frac{\partial n_2}{\partial n_1^H}$ and the marginal cost c as their difference must be sufficiently large. For $\frac{\partial n_2}{\partial n_1^H} > 0$, the expression $\left(\tilde{p}^* - \alpha \cdot (\tilde{n}_1^{L*} + \tilde{n}_1^{H*}) \right) \cdot \frac{\partial n_2}{\partial n_1^H}$ covers the (net) marginal profit that arises from the indirect impact of an increase in n_1^H on the demand of market side 2 agents. In case that this expression is very large, it implies that an additional unit n_1^H is particularly profitable due to the presence

of the opposite market side. If, on the other hand, c is very small, there is a strong incentive for the monopolist to choose n_1^H (as well as n_1^L) as large as possible. However, the platform operator is facing a difficult trade-off: For $\alpha > 0$, any increase of n_1^L and n_1^H reduces the utility of market side 1 agents, but their participation is crucial in order to exploit the high marginal profits from market side 2. Therefore, the platform operator has to compensate both types of agents for their utility losses by reducing the bundle-specific payment. As we have found that $\tilde{n}_1^{H*} > \tilde{n}_1^{L*}$, we know that high-demand agents on market side 1 are facing a larger extent of utility reduction in the optimum, since the interaction with market side 2 agents is assumed to depend on the chosen bundle. Hence, in order to respect the participation constraints, the price reduction for the H-type bundle will exceed the one for the L-type bundle. In case that (24) holds, this process leads to the surprising result $\tilde{t}_H^* < \tilde{t}_L^*$. The economic intuition is as follows: As per usual, high-demand agents obtain more utility from joining the platform, i.e. it holds that $\theta_H \cdot u(\tilde{n}_1^{H*}) > \theta_L \cdot u(\tilde{n}_1^{L*})$. At the same time they are facing more disutility from the presence of market side 2 agents, i.e. we have that $\tilde{n}_1^{H*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*) > \tilde{n}_1^{L*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*)$. If

$$\theta_H \cdot u(\tilde{n}_1^{H*}) - \theta_L \cdot u(\tilde{n}_1^{L*}) < \tilde{n}_1^{H*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*) - \tilde{n}_1^{L*} \cdot \alpha \cdot n_2(\tilde{n}_1^{L*}, \tilde{n}_1^{H*}, \tilde{p}^*),$$

which holds if (24) is satisfied, the platform operator has to compensate high-demand agents by choosing $\tilde{t}_H^* < \tilde{t}_L^*$.

Optimization under incomplete information

As was already discussed in Section 3, the monopolist has to respect the incentive constraints and the participation constraints when maximizing her profit under incomplete information. Hence, the corresponding (Kuhn-Tucker) optimization problem is described by

$$(25) \quad \max_{t_L, t_H, n_1^L, n_1^H, p} \Pi = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p)$$

s.t.

$$(25a) \quad \theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0,$$

$$(25b) \quad \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq 0,$$

$$(25c) \quad \theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq \theta_L \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H,$$

$$(25d) \quad \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H \geq \theta_H \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L.$$

Respecting $\theta_H > \theta_L$, we find by using equations (25a) and (25d) that

$$\begin{aligned} \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H &\geq \theta_H \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \\ &> \theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L \geq 0 \Rightarrow \theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H > 0. \end{aligned}$$

Hence, the participation constraint for high-demand agents can be disregarded as it is never binding. The resulting Kuhn-Tucker optimization problem is therefore given by

$$\begin{aligned} (26) \quad \max_{t_L, t_H, n_1^L, n_1^H, p, \lambda_1, \lambda_2, \lambda_3} \quad & L = t_L + t_H - c \cdot n_1^L - c \cdot n_1^H + p \cdot n_2(n_1^L, n_1^H, p) \\ & + \lambda_1 \cdot [\theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L] \\ & + \lambda_2 \cdot [\theta_L \cdot u(n_1^L) - n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_L - \theta_L \cdot u(n_1^H) + n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) + t_H] \\ & + \lambda_3 \cdot [\theta_H \cdot u(n_1^H) - n_1^H \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) - t_H - \theta_H \cdot u(n_1^L) + n_1^L \cdot \alpha \cdot n_2(n_1^L, n_1^H, p) + t_L] \end{aligned}$$

with the first-order conditions

$$(27) \quad \frac{\partial L}{\partial t_L} = 1 - \lambda_1 - \lambda_2 + \lambda_3 = 0,$$

$$(28) \quad \frac{\partial L}{\partial t_H} = 1 + \lambda_2 - \lambda_3 = 0,$$

$$(29) \quad \frac{\partial L}{\partial n_1^L} = (\lambda_1 + \lambda_2) \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \lambda_3 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^L} - \alpha \cdot n_2(\cdot) \cdot (\lambda_1 + \lambda_2 - \lambda_3) + \Psi \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(30) \quad \frac{\partial L}{\partial n_1^H} = \lambda_3 \cdot \theta_H \cdot \frac{\partial u}{\partial n_1^H} - \lambda_2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^H} - \alpha \cdot n_2(\cdot) \cdot (\lambda_3 - \lambda_2) + \Psi \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(31) \quad \frac{\partial L}{\partial p} = n_2(\cdot) + \Psi \cdot \frac{\partial n_2}{\partial p} = 0,$$

where $\Psi = (p - \alpha \cdot (n_1^L \cdot (\lambda_1 + \lambda_2 - \lambda_3) + n_1^H \cdot (\lambda_3 - \lambda_2)))$.

Taking into account that the Kuhn-Tucker conditions require $\lambda_m \geq 0$, $m = 1, 2, 3$, equations (27) and (28) imply that in the optimum $\lambda_1 = 2$, $\lambda_2 = 0$, and $\lambda_3 = 1$ must hold. Hence, equations (29) to (31) become

$$(32) \quad \underbrace{\frac{\partial L}{\partial n_1^L} = 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L}}_{< \theta_L \cdot \partial u / \partial n_1^L} - \alpha \cdot n_2(\cdot) + (p - \alpha \cdot (n_1^L + n_1^H)) \cdot \frac{\partial n_2}{\partial n_1^L} - c = 0,$$

$$(33) \quad \frac{\partial L}{\partial n_1^H} = \theta_H \cdot \frac{\partial u}{\partial n_1^H} - \alpha \cdot n_2(\cdot) + (p - \alpha \cdot (n_1^L + n_1^H)) \cdot \frac{\partial n_2}{\partial n_1^H} - c = 0,$$

$$(34) \quad \frac{\partial L}{\partial p} = \underbrace{n_2(\cdot)}_{>0} + \underbrace{(p - \alpha \cdot (n_1^L + n_1^H))}_{>0(1)} \cdot \underbrace{\frac{\partial n_2}{\partial p}}_{<0} = 0.$$

Obviously, we find from equation (34) that an interior solution still strictly requires that $(p - \alpha \cdot (n_1^L + n_1^H)) > 0$. Additionally assuming that both types of agents on market side 1 are served in the optimum, equations (32) - (34) determine the unique profit-maximizing solution $(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**})$.

Since $\theta_H > \theta_L$ and $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H}$, we find by analyzing equations (32) and (33) that $\frac{\partial L}{\partial n_1^H} > \frac{\partial L}{\partial n_1^L}$, which implies that the optimal level of platform access for high-demand agents still exceeds the one for low-demand agents, i.e. we have $\tilde{n}_1^{H**} > \tilde{n}_1^{L**}$. From the Kuhn-Tucker constraints we can conclude that there is still no consumer surplus for the L-type, while the H-type is indifferent between both bundles and enjoys a strictly positive consumer surplus. These results are consistent with our findings from the previous section.. As we know that (25a) as well as (25d) are binding in the optimum, we find that the optimal payments are given by

$$\begin{aligned}\tilde{t}_L^{**} &= \theta_L \cdot u(\tilde{n}_1^{L**}) - \tilde{n}_1^{L**} \cdot \alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**}) \\ \tilde{t}_H^{**} &= \theta_H \cdot u(\tilde{n}_1^{H**}) - \tilde{n}_1^{H**} \cdot \alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**}) - \theta_H \cdot u(\tilde{n}_1^{L**}) + \theta_L \cdot u(\tilde{n}_1^{L**}).\end{aligned}$$

Hence, the price differential is expressed by

$$(35) \quad \tilde{t}_H^{**} - \tilde{t}_L^{**} = \theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**}) + \alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**}) \cdot (\tilde{n}_1^{L**} - \tilde{n}_1^{H**}).$$

In the benchmark case of complete information we found that under specific restrictions the L-type bundle was more expensive than the H-type bundle. However, in case of asymmetric information, the monopolistic platform operator is not able to sell type-specific take-it-or-leave-it offers to agents on market side 1, which potentially influences our results. The corresponding findings under incomplete information are summarized by Proposition 3:

Proposition 3: In case of incomplete information, it holds in the optimum that the profit-maximizing tariff for the H-type bundle is strictly larger than the payment for the L-type bundle if the presence of market side 2 agents exerts a nonnegative externality on both types of agents on market side 1, i.e. for $\alpha \leq 0$. For $\alpha > 0$, the results are ambiguous, which implies that, under specific conditions, the price level of the L-type bundle exceeds the payment for the H-type bundle.

Proof: First, we consider the case of positive network effects on market side 1, i.e. $\alpha < 0$. Then, respecting $\tilde{n}_1^{H**} > \tilde{n}_1^{L**}$, we find by using equation (35) that

$$\tilde{t}_H^{**} - \tilde{t}_L^{**} = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**})}_{<0} \cdot \underbrace{(\tilde{n}_1^{L**} - \tilde{n}_1^{H**})}_{<0} > 0.$$

For $\alpha = 0$, equation (35) simplifies to

$$\tilde{t}_H^{**} - \tilde{t}_L^{**} = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**})}_{>0} > 0,$$

which proves the first statement that is contained in Proposition 3.

Now, suppose that $\alpha > 0$, which implies the presence of a negative indirect network externality exerted on market side 1 agents. In this case, we find from (35) that

$$\tilde{t}_H^{**} - \tilde{t}_L^{**} = \underbrace{\theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**})}_{>0} \cdot \underbrace{(\tilde{n}_1^{L**} - \tilde{n}_1^{H**})}_{<0},$$

yielding ambiguous results, i.e. $\tilde{t}_H^{**} - \tilde{t}_L^{**}$ can have any sign or is equal to zero. Showing that $\tilde{t}_H^{**} - \tilde{t}_L^{**} < 0$ is feasible in the optimum, requires to verify that

$$\underbrace{\theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**})}_{>0} + \underbrace{\alpha \cdot n_2(\tilde{n}_1^{L**}, \tilde{n}_1^{H**}, \tilde{p}^{**})}_{>0} \cdot \underbrace{(\tilde{n}_1^{L**} - \tilde{n}_1^{H**})}_{<0} < 0$$

is covered by the first-order conditions. We start the proof by assuming that $\theta_H \approx \theta_L$. Hence, we have in the optimum that $\tilde{n}_1^{L**} = \tilde{n}_1^{H**} - \varepsilon$ with $\varepsilon > 0$ being very small. By approximation, it holds that

$$\theta_H \cdot u(\tilde{n}_1^{H**}) - \theta_H \cdot u(\tilde{n}_1^{L**}) = \theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} \cdot (\tilde{n}_1^{H**} - \tilde{n}_1^{L**}) = \theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} \cdot \varepsilon.$$

Then, using equation (35) we find that $\tilde{t}_H^{**} - \tilde{t}_L^{**}$ can be described by

$$\tilde{t}_H^{**} - \tilde{t}_L^{**} = \theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} \cdot \varepsilon + \alpha \cdot n_2(\cdot) \cdot (-\varepsilon) = \varepsilon \cdot \left(\theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} - \alpha \cdot n_2(\cdot) \right),$$

which allows us to conclude that

$$\tilde{t}_H^{**} - \tilde{t}_L^{**} < 0 \Leftrightarrow \theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} - \alpha \cdot n_2(\cdot) < 0 \Leftrightarrow \theta_H \cdot \left. \frac{\partial u}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}} < \alpha \cdot n_2(\cdot).$$

By analyzing (33) we find that this inequality is satisfied in the optimum if it holds that

$$(36) \quad \underbrace{(\tilde{p}^{**} - \alpha \cdot (\tilde{n}_1^{L**} + \tilde{n}_1^{H**}))}_{>0} \cdot \underbrace{\left. \frac{\partial n_2}{\partial n_1^H} \right|_{\tilde{n}_1^{H**}}}_{>0(l)} - \underline{c} > 0,$$

which strictly requires positive indirect network effects on market side 2, i.e. $\frac{\partial n_2}{\partial n_1^L} = \frac{\partial n_2}{\partial n_1^H} > 0$.

(q.e.d.)

Proposition 3 shows that our results from the benchmark case of complete information prevail under incomplete information: It is still a feasible solution in the optimum that the L-type bundle is more expensive than the bundle for high-demand agents. This is not surprising as

we know that the H-type enjoys a strictly positive consumer surplus under incomplete information, while the monopolist extracts the entire consumer surplus in case of perfect price discrimination. Hence, in order to make high-demand agents better off than under complete information, the monopolistic platform operator *ceteris paribus* has an incentive to increase the H-type's utility in the optimum by choosing \tilde{t}_H^{**} below the level of the benchmark case. Therefore, as we already know that $\tilde{t}_H^* - \tilde{t}_L^* < 0$ is feasible under complete information, it is not surprising that this outcome prevails under incomplete information.

Comparison with the benchmark case

We will close the analysis by comparing our findings from the case of asymmetric information to the benchmark case of complete information. Proposition 4 contains the corresponding results.

Proposition 4:

Restricting the analysis to $\frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p} < \alpha < -\frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p}$ for the case of positive network effects on market side 2, i.e. $\frac{\partial n_2}{\partial n_1^i} > 0$, $i = H, L$, and $-\frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p} < \alpha < \frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p}$ for $\frac{\partial n_2}{\partial n_1^i} < 0$, we find that under incomplete information, the optimal extent of platform access for low-demand agents is strictly smaller than the benchmark level, i.e. it holds that $\tilde{n}_1^{L**} < \tilde{n}_1^{L*}$. In addition, the profit-maximizing amount of platform access for high-demand agents is also negatively affected under incomplete information. Hence, we find that $\tilde{n}_1^{H**} < \tilde{n}_1^{H*}$. Given the restrictions above, this result is robust with respect to the sign of the network effect on market side 2.

Proof: We start by considering the case of $\frac{\partial n_2}{\partial n_1^i} > 0$. Since inequality (16) is still satisfied, it is easy to show by using equations (20) and (32) that the monopolistic platform operator under incomplete information *ceteris paribus* chooses a smaller amount of platform access for the L-type bundle. This effect induces a change in the demand of market side 2 agents, which in turn influences the profit-maximizing price p^{**} that is implicitly given by equation (34). Differentiating (34) with respect to n_1^L , while respecting that $\frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p} < \alpha < -\frac{\partial n_2}{\partial n_1^i} / \frac{\partial n_2}{\partial p}$ and $\frac{\partial^2 n_2}{\partial p \partial n_1^L} = 0$, yields

$$\frac{\partial^2 L}{\partial p \partial n_1^L} = \frac{\partial n_2}{\partial n_1^L} + \underbrace{\left(p - \alpha \cdot (n_1^L + n_1^H) \right)}_{=0} \cdot \frac{\partial^2 n_2}{\partial p \partial n_1^L} - \alpha \cdot \frac{\partial n_2}{\partial p} > 0,$$

which immediately implies that $p^{**} < p^*$. This reduction has an additional effect on the optimal amount of platform access for L-type agents. By solving (34) for $n_2(\cdot)$ we obtain

$$n_2(\cdot) = -\left(p - \alpha \cdot (n_1^L + n_1^H) \right) \cdot \frac{\partial n_2}{\partial p},$$

so that equation (32) can be reformulated as

$$\frac{\partial L}{\partial n_1^L} = 2 \cdot \theta_L \cdot \frac{\partial u}{\partial n_1^L} - \theta_H \cdot \frac{\partial u}{\partial n_1^L} + \underbrace{\left(p - \alpha \cdot (n_1^L + n_1^H) \right)}_{>0} \cdot \underbrace{\left(\alpha \cdot \frac{\partial n_2}{\partial p} + \frac{\partial n_2}{\partial n_1^L} \right)}_{>0} - c = 0.$$

Taking into account that $\frac{\partial n_2}{\partial n_1^L} / \frac{\partial n_2}{\partial p} < \alpha < -\frac{\partial n_2}{\partial n_1^H} / \frac{\partial n_2}{\partial p}$, this allows us to conclude that $\tilde{n}_1^{L**} < \tilde{n}_1^{L*}$. Since p^{**} simultaneously enters equation (33), it can be analogously shown that $\tilde{n}_1^{H**} < \tilde{n}_1^{H*}$. For $\frac{\partial n_2}{\partial n_1^H} < 0$, the proof follows the same logic, but requires $-\frac{\partial n_2}{\partial n_1^H} / \frac{\partial n_2}{\partial p} < \alpha < \frac{\partial n_2}{\partial n_1^L} / \frac{\partial n_2}{\partial p}$.
(*q.e.d.*)

Proposition 4 confirms our findings from Section 3 and shows that our results are still holding in case of bundle-specific interaction. The logic behind our findings remains the same as in Section 3: Under incomplete information, the monopolist *ceteris paribus* has an incentive to reduce the optimal extent of platform access for low-demand agents below the benchmark level, in order to make this bundle less attractive for high-demand agents. Thus, the firm is able to extract additional consumer surplus from the H-type on market side 1. However, for $\frac{\partial n_2}{\partial n_1^L} > 0$ the reduction of n_1^L has a negative influence on the demand of market side 2 agents, which additionally reduces the marginal profit from an additional unit of n_1^L . Hence, there is a second effect that reduces the optimal amount of platform access for low-demand agents on market side 1. At the same time this effect has an impact on the marginal profit that is generated from an additional unit n_1^H . Since each unit n_1^H becomes less profitable due to the reduced demand on market side 2, the profit-maximizing extent of platform access that is provided to the H-type is also strictly below the benchmark level.

5. Conclusions and Implications

The present paper provides a positive analysis of second-degree price discrimination on a monopolistic two-sided market. We found that many of the results from the equivalent problem on one-sided markets are still valid in our two-sided setting: The extent of platform access (which may well be interpreted as quality) for low-demand agents is strictly reduced under incomplete information, in order to induce the well-known self-selection process among agents on market side 1, while allowing the monopolist to extract additional consumer surplus from high-demand agents. In addition, in the optimum agents with low valuation are still left without any consumer surplus, whereas high-demand agents are in any case indifferent between the two bundles, while enjoying an information rent.

However, the paper contributes to the existing literature by revealing some important differences in relation to the second-degree price discrimination problem on one-sided markets: In Section 3, we found that the famous “no-distortion-at-the-top”-rule from one-sided markets does not prevail in our two-sided framework, since our analysis yielded that $n_1^{H**} < n_1^{H*}$, i.e. under incomplete information, the extent of platform access for the H-type is strictly below the level under complete information. In the subsequent analysis of Section 4 it was also shown that this result is robust with respect to the model specification of bundle-specific interaction. This result implies that the monopolistic optimization problem is more complex in the presence of network externalities as the ceteris paribus profit-enhancing reduction of n_1^L has an impact on the demand of market side 2 agents, which in turn (negatively) affects the marginal profit generated from an additional unit n_1^H .

The analysis of Section 4 implied another important result: In case of bundle-specific interaction, it is a feasible profit-maximizing solution that the bundle for the L-type is more expensive than the one for the H-type, which contradicts the findings from one-sided markets. This surprising result holds for the case of complete information and prevails under incomplete information. Interpreting these findings sheds some new light on the examples provided in Section 1. For instance, recall the case of German newspaper “Bild” that offers exclusive print media content on the internet free of charge, but in combination with advertising. Since the online content is identical to the content of the printed newspaper, there is no quality reduction, but a significant amount of advertising consisting of numerous banner ads and links. Intuitively, one would expect that offering the online content “for free” targets low-demand agents, since they do not have to pay for being able to read the same content that is offered in the print medium. However, respecting the results of Propositions 2 and 3, it may well be the case that the combination of free online access and higher advertising levels is a designated bundle for high-demand agents. This is supported by the following consideration: If we assume that high-demand agents on a newspaper market have a strong preference for receiving exclusive news as soon as possible, the results of our model would strictly require publishing any information on the internet at first. This can indeed be observed as the articles of “Bild” are usually published on the internet the night before the printed newspaper is sold.

Finally, it has to be mentioned that the focus of our analysis is exclusively positive, so that we are not able to draw any normative conclusions with respect to the welfare aspects of the second-degree price discrimination problem on a two-sided market. Extending the analysis by

welfare considerations or a second group of agents on market side 2 remains the task for further research.

References

Anderson S.P. and Coate, S. (2005): “Market provision of broadcasting: A welfare analysis”, *Review of Economic Studies*, 72(4), 947-972.

Armstrong, M. (2006a): “Recent developments in the economics of price discrimination”, In: *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress of the Econometric Society, Volume 2*, by: R. Blundell, W. Newey and T. Persson (eds.), Cambridge University Press, 97-141.

Armstrong, M. (2006b): “Competition in two-sided markets”, *Rand Journal of Economics*, 37(3), 668-691.

Caillaud, B. and Jullien, B. (2003): “Chicken and egg: Competition among intermediation service providers”. *Rand Journal of Economics*, 34(2), 309-328.

Gabszewicz, J. J., Laussel, D. and Sonnac, N. (2004): “Programming and advertising competition in the broadcasting industry”. *Journal of Economics & Management Strategy*, 13(4), 657-669.

Gal-Or, E. and Dukes, A. (2003): “Minimum differentiation in commercial media markets”. *Journal of Economics and Management Strategy*, 12(3), 291-325.

Liu, Q. and Serfes, K. (2010): “Price discrimination in two-sided markets”. Working Paper, University of Oklahoma.

Maskin, E. and Riley, J. (1984): “Monopoly with incomplete information”. *Rand Journal of Economics*, 15(2), 171-196.

Mussa, M. and Rosen, S. (1978): “Monopoly and product quality”. *Journal of Economic Theory*, 18(2), 301-317.

Peitz, M. and Valletti, T. M. (2008): “Content and advertising in the media: pay-tv versus freeto-air”. *International Journal of Industrial Organization*, 26(4), 949-965.

Reisinger, M. (2010): “Unique equilibrium in two-part tariff competition between two-sided platforms”. Working Paper, University of Munich.

Rochet, J.-C. and Tirole, J. (2003): “Platform competition in two-sided markets”, *Journal of the European Economic Association*, 1(4), 990-1029.

Rochet, J.-C. and Tirole, J. (2006): “Two-sided markets: a progress report”, *RAND Journal of Economics*, 37(3), 645-667.

Spence, M. (1977): “Nonlinear prices and welfare”. *Journal of Public Economics*, 8(1), 1-18.

Spulber, D.F. (1993): “Monopoly pricing”. *Journal of Economic Theory*, 59(1), 222-234.

Stiglitz, J. E. (1977): “Monopoly, non-linear pricing and imperfect information: The insurance market”. *Review of Economic Studies*, 44(3), 407-430.

Stole, L. A. (1995): “Nonlinear pricing and oligopoly”. *Journal of Economics and Management Strategy*, 4(4), 529-562.

Stole, L. A. (2007): “Price discrimination and competition”. In: *Handbook of Industrial Organization, Volume 3*, by: M. Armstrong, and R. Porter (eds.), Elsevier, Amsterdam: North-Holland, 2221-2299.